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1 Efficiency of attack strategies on complex model and real-world networks

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13

14 Abstract

15 We investigated the efficiency of attack strategies to network nodes when targeting several
16 complex model and real-world networks. We tested 5 attack strategies, 3 of which were
17 introduced in this work for the first time, to attack 3 model networks (Erdos and Renyi,
18 Barabasi and Albert preferential attachment network, and scale-free network
19 configuration models) and 3 real networks (Gnutella peer-to-peer network, email network
20 of the University of Rovira i Virgili, and immunoglobulin interaction network). Nodes
21 were removed sequentially according to the importance criterion defined by the attack
22 strategy, and we used the size of the largest connected component (*LCC*) as a measure of
23 network damage. We found that the efficiency of attack strategies (fraction of nodes to be
24 deleted for a given reduction of *LCC* size) depends on the topology of the network,
25 although attacks based on either the number of connections of a node or betweenness
26 centrality were often the most efficient strategies. Sequential deletion of nodes in
27 decreasing order of betweenness centrality was the most efficient attack strategy when
28 targeting real-world networks. The relative efficiency of attack strategies often changed
29 during the sequential removal of nodes, especially for networks with power-law degree
30 distribution.

31

32 1. Introduction

33 The resilience of real-world complex networks, such as Internet, electrical power grids,
34 airline routes, ecological and biological networks [1-6] to “node failure” (i.e. node
35 malfunctioning or removal) is a topic of fundamental importance for both theoretical and
36 applied network science. Node failure can cause the fragmentation of the network, which
37 has consequences in terms of system performance, properties, and architecture, such as
38 transportation properties, information delivery efficiency and the reachability of network
39 components (i.e. ability to go from node of the network to another) [3].

40 Several studies [3,7,8,9] have investigated the resilience of model networks using a
41 number of “attack strategies”, i.e. a sequence of node removal according to certain
42 properties of the nodes [2,3,7]. A widely-applied attack strategy consists in first ranking
43 the nodes with respect to an importance criterion (e.g. number of connections or some
44 measure of centrality) and then remove the nodes sequentially from the most to the least
45 important according to the chosen criterion until the network either becomes disconnected
46 or loses some essential qualities [3,10]. However, little is known on how the efficiency of
47 attack strategies (i.e. the fraction of nodes to be deleted for a given change in the network)
48 varies when considering different real-world and model networks.

49 In this context, an underappreciated problem is how the relative efficiency of attack
50 strategies may change during the attack to the network. For example, an attack strategy
51 might be more efficient when the targeted (i.e. under attack) network is still pristine, while
52 other strategies may be more efficient when the network has already been fragmented and
53 some of its properties have been compromised. Testing the efficiency of the different
54 attack strategies when targeting different networks may also allow to identify the most

55 important nodes for network functioning, and therefore which nodes should be primarily
56 protected, as in the case of computer [11] or ecological networks [6,12-14], or removed, as
57 in the case of immunization/disease networks [15].

58 In this work, we test the efficiency of both well-known attack strategies and new strategies
59 introduced for the first time in this paper when targeting either model or real-world
60 networks. We used the size of the largest connected component (*LCC*) (i.e. the largest
61 number of nodes connected among them in the network, [2]) as a measure of network
62 damage. We found for model networks that the best strategy to reduce the size of the *LCC*
63 depended on the topology of the network that was attacked. For real-world networks, the
64 removal of nodes using betweenness centrality as importance criterion was consistently
65 the most efficient attack strategy. For some networks, we found that an attack strategy can
66 be more efficient than others up to a certain fraction of nodes removed, but other attack
67 strategies can become more efficient after that fraction of nodes has been removed.

68 2.Methods

69 2.1 Attack strategies

70 We attacked the networks by sequentially removing nodes following some importance
71 criteria. We compared the efficiency of a pool of attacks strategies, some of which have
72 been already described in the literature while others are introduced in this work for the
73 first time.

74 Most of the analyses on the robustness of network have investigated the effect of removing
75 nodes according to their rank (i.e. number of links of the node) or some measures of
76 centrality [3,10,16]. In this work, we introduce new attack strategies that focus entirely or

77 in part on less local properties of a node, in particular its number of second neighbors, as
78 explained in detail below.

79 Several indexes and measures have been proposed in order to describe network damage.
80 We use the size of the largest connected component (*LCC*), i.e. the size of the largest
81 connected sub-graph in the network [2,3], as a measure of network damage during the
82 attack, where a faster decrease in the size of the *LCC* indicates a more efficient attack
83 strategy. In order to compare attack strategies across networks, we normalized *LCC* size at
84 any point during the attack with respect to the starting *LCC* size, i.e. the number of nodes
85 in the *LCC* before the attack.

86 For each attack strategy, we applied both the recalculated and non-recalculated method.
87 With the recalculated method, the property of the node relevant for the attack strategy
88 (e.g. number of links) was recalculated after each node removal. On the other hand, when
89 applying the non-recalculated method the property of the node was measured before the
90 first node removal and was not updated during the sequential deletion of nodes. With q
91 we indicate the fraction of nodes removed during the sequential removal of nodes. An
92 attack strategy is less efficient than another when a higher q to reduce the *LCC* to zero (or
93 any other size).

94 In this work, we used 2 attack strategies that have already been described in the literature.
95 *First-degree neighbors (First)*: nodes are sequentially removed according to the number of
96 first neighbors of each node (i.e. node rank). In the case of ties (i.e. nodes with the same
97 rank), the sequence of removal of nodes is randomly chosen. *Nodes betweenness centrality*
98 (*Bet*): nodes are sequentially removed according to their betweenness centrality, which is

99 the number of shortest paths from all vertices to all others that pass through that node
100 [3,17].

101 We introduced in the present work the following new attack strategies. *Second-degree*
102 *neighbors (Sec)*: nodes are sequentially removed according to the number of second
103 neighbors of each node. Second neighbors of node j are nodes that have a node in common
104 with - but are not directly connected to - node j . *First + Second neighbors (F+S)*: nodes are
105 deleted according to the sum of first and second neighbors of each node. *Combined first and*
106 *second degree (Comb)*: nodes are removed according to their rank. In the case of ties, nodes
107 are removed according to their second degree.

108 For all attack nodes were sequentially removed from most to least connected. In the case of
109 *Bet*, nodes were sequentially removed from higher to lower betweenness centrality. For
110 each network described in Section 2.2, we tested the relative efficiency of the five attack
111 strategies in reducing the LCC to zero. In addition, we tested whether the relative
112 efficiency of attack strategies changed along the removal sequence, i.e. whether an attack
113 strategies was less efficient than another at the beginning of the attack, but more efficient
114 after a fraction q of nodes was removed.

115 2.2 Networks

116 We tested the attack strategies described in Section 2.2 on 3 types of model networks and 3
117 real world networks.

118 The 6 networks are undirected and unweighted graphs in which nodes are connected by
119 links or edges, and rank k of a node is the number of links of that node. Each link may
120 represent several real world interactions. For instance, in social networks links between

121 nodes represent interactions between individuals or groups, such as co-authorship in
122 scientific publications or friendship [2]. In cellular networks, nodes are chemicals species
123 connected by chemical reactions [18], while in ecological networks links describe the
124 trophic interactions between species or group of species, e.g. the energy and matter
125 passing from prey to predator [6,14,19,20].

126 2.2.1 Model networks

127 We tested the attack strategies on (i) Erdos and Renyi graphs [21], (ii) Barabasi and Albert
128 preferential attachment networks [2], and (iii) scale-free network configuration models
129 [22]. For each model network, we tested the efficiency of attack strategies on networks of
130 different size, as explained below. Since each model network is a random realization of the
131 network-generating mechanism, we tested the attack strategies on 50 random realizations
132 of each model network used the mean across replicates of the normalized *LCC* size at each
133 fraction q of nodes removed as a measure of network damage. We observed a small
134 variation of *LCC* size at each fraction q of nodes removed across different realizations of
135 networks, thus the mean *LCC* size across replicates well represented the overall behavior
136 of the attack strategy.

137 The Erdos and Renyi (*ER*) model generates a random graph with N nodes connected by L
138 links, which are chosen randomly with an occupation probability p from $L_{\max} = N(N-1)/2$
139 possible links, i.e. p is the proportion of realized links from L_{\max} . The expected number of
140 links is $\langle L \rangle = (N^2 p)/2$ and the expected rank of a node is $\langle k \rangle = Np$. The random graph can
141 be defined by the number of nodes N and the occupation probability p , i.e. $ER(N,p)$ [21].

142 We analyzed *ER* graphs with different values of N and p , specifically: $ER(N = 500, p =$
143 $0.008)$, $ER(1\ 000, 0.004)$, $ER(10\ 000, 0.0004)$.

144 The Barabasi and Albert preferential attachment network (*BA*) is created starting from few
 145 isolated nodes and by then growing the network by adding new nodes and links [2]. At
 146 each step in the creation of the network, one node and m outgoing links from the new
 147 node are added to the network. The probability θ that the new node will be connected to
 148 node i already in the network is function of the degree k_i of node i , such that

$$149 \quad \theta(k_i) = k_i / \sum_{j=N}^{j=1} k_j \quad (\text{i.e. preferential attachment, since } k \text{ more connected nodes are more likely}$$

150 to be connected to the new node) [2]. The *BA* network is defined by parameters N and m .
 151 We built *BA* scale free networks with parameters $BA(N=500, m = 2)$, $BA(1\ 000, 2)$, $BA(10$
 152 $000, 2)$.

153 We created networks with power-law degree distribution using the configuration model
 154 for generalized random graphs [2,22]. This model is defined as follows. A discrete degree
 155 distribution $P(K = k) = k^{-\alpha}$ is defined, such that $P(k)$ is the proportion of nodes in the
 156 network having degree k . The maximum node degree k_{\max} is equal to N , where N is the
 157 number of nodes. The domain of the discrete function $P(k)$ becomes $(1, k_{\max})$. We generated
 158 the degree sequence of the nodes by randomly drawing N values k_1, \dots, k_n from the degree
 159 distribution. Then, for each node i we assigned a link with node j with probability
 160 $P(k_i)P(k_j)$. A scale free configuration model network is defined by parameters N , α and $\langle k \rangle$.
 161 We analyzed scale free network with parameters $CM(N = 500, \alpha = 2.5, \langle k \rangle = 3.8)$, $CM(1$
 162 $000, 2.5, 3.8)$, $CM(10\ 000, 2.5, 3.9)$.

163 2.2.2 Real world networks

164 We tested the attack strategies on the following real-world networks: (i) The Gnutella P2P
 165 (peer-to-peer) network (*Gnutella*) [24], (ii) the email network of the University Rovira i

166 Virgili (URV) in Tarragona, Spain (*Email*) [25], and (iii) the immunoglobulin interaction
167 network (*Immuno*) [26]. Nodes of *Gnutella* ($N=8\ 846$, $L=31\ 839$) represent hosts in the peer-
168 to-peer network, while links represent connections between the hosts [24]. *E-mail* ($N=1$
169 134 , $L=10\ 902$) provides an example of the flow of information within a human
170 organization [25]. *Immuno* is the undirected and connected graph of interactions in the
171 immunoglobulin protein ($N = 1\ 316$, $L = 6\ 300$) where nodes represent amino acids, and
172 two amino acids are linked if they interact in the immunoglobulin protein [26].

173 3. Results

174 3.1 Non-Recalculated method

175 3.1.1 Model networks (Fig. 1 and Fig. A1)

176 **ER:** For all sizes of networks, the 5 attack strategies were equally efficient in reducing the
177 size of the LCC up to $q \sim 0.2$. For $q > 0.2$, *First* was the most efficient strategy to reduce the
178 size of the LCC to 0.

179 **CM:** For $N = 500$, *Comb* was the most efficient strategy early in the removal sequence.,
180 while *First* became the most efficient strategy for $q > 0.1$. For $N = 1\ 000$, *Comb*, *Bet*, and *First*
181 had the same efficiency. For $N = 10\ 000$, *Comb*, *Bet*, and *First* were equally efficient up to q
182 $= 0.1$, while for $q > 0.1$ *First* was the most efficient strategy.

183 **BA:** For $N = 500$, *First*, *Comb* and *Bet* were equally efficient in reducing the size of the LCC.
184 For bigger networks, *First*, *Comb* and *Bet* were equally efficient up to $q = 0.8$ ($N = 1\ 000$)
185 and $q = 0.5$ ($N = 10\ 000$). Then, *Bet* became more efficient than *First* and *Comb*.

186 3.1.2 Real-world networks (Fig. 2 and Fig. A2)

187 **Email:** *Bet* was the most efficient strategy to reduce LCC up to $q \approx 0.3$. For greater
188 fractions of nodes removed, *First* and *Comb* were slightly more efficient than *Bet*.

189 **Immuno:** *Bet* was distinctly more efficient than other strategies up to $q = 0.55$. For $q > 0.55$,
190 all strategies were equally efficient.

191 **Gnutella:** *Bet* was the most efficient attack strategy.

192 3.2 Recalculated method

193 3.2.1 Model networks (Fig. 3 and Fig. A3)

194 **ER:** *First* and *Comb* were the most efficient strategies to reduce the LCC up to $q \approx 0.2$. For q
195 > 0.2 , *Bet* became more efficient than *First*. *Sec* was the least efficient strategy.

196 **CM:** *Comb* was the most efficient strategy up to $q \approx 0.1$. For $q > 0.1$, *Bet* was the most
197 efficient strategy, while *Sec* was the least efficient strategies.

198 **BA:** *Comb* was the most efficient strategy up to $q \approx 0.1$. *First*, *F+S* and *Bet* attack induced a
199 slightly slower decrease in LCC size. For $q > 0.1$, *Bet* became the most efficient strategy. *Sec*
200 was the least efficient strategy.

201 3.2.2 Real-world networks (Fig. 4 and Fig. A4)

202 **Email:** All attack strategies were equally efficient up to $q = 0.12$. For $q > 0.12$, *Bet* was the
203 most efficient attack strategy.

204 **Immuno:** *Bet* was largely the most efficient attack strategy.

205 **Gnutella:** All attack strategies were equally efficient up to $q = 0.1$. For $q > 0.1$, *Bet* was the
206 most efficient attack strategy.

207 4. Discussion

208 We discuss the following main results of our work: (i) attacks were largely more efficient
209 with the recalculated than with the non-recalculated method; (ii) the efficiency of attack
210 strategies on model networks depended on network topology; (iii) the sequential removal
211 of nodes according to their betweenness centrality was the most efficient attack to real-
212 world networks; (iv) for some networks, the relative efficiency of attack strategies changed
213 during the removal sequence.

214 We found that the recalculated method provided more efficient attacks than the non-
215 recalculated method, i.e. for a given fraction of nodes removed from the network, a larger
216 reduction of LCC was obtained with the recalculated method. This result confirms the
217 findings of other analyses on robustness of networks [2,3], which found that updated
218 information on the topology of the system after each removal allowed for more efficient
219 attacks to networks.

220 However, non-recalculated attack strategies are implemented in various relevant settings
221 and are equivalent in practice to the simultaneous removal of nodes, as it happens in the
222 case of vaccination campaigns (i.e. the strategy is vaccinating at the same time nodes of
223 the contact network with the highest probability of acquiring or transmitting the disease)
224 or attacks to computer networks [11].

225 For model networks, the efficiency of the attack strategies depended on network topology.
226 In the case of networks with power-law degree distribution, the efficiency of the attack
227 strategies depended also on network size. Across all model networks and considering both
228 the non-recalculated and recalculated methods, attack strategies based on either node
229 betweenness centrality or node rank were the most efficient ones. However, the sequential

230 deletion of nodes according to their betweenness centrality was consistently the most
231 efficient attack strategy to real-world networks, with the only exception of the attack to the
232 *Email* network with the non-recalculated method. While in some cases *Bet* was only
233 slightly more efficient than other strategies in reducing the size of the largest connected
234 component, in others *Bet* was largely the most efficient strategy. For example, in the
235 immunoglobulin interaction network, deleting a very small fraction of nodes with high
236 betweenness centrality reduced the size of the normalized *LCC* of more than 60% using
237 either the recalculated and non-recalculated method, while - for the same fractions of
238 nodes removed - other attack strategies caused only a 1-5% reduction in *LCC* size.

239 Betweenness centrality describes how “central” a node is in the network by considering
240 the fraction of shortest paths that pass through that node [17]. Nodes with betweenness
241 centrality greater than 0 play a major role in connecting areas of the network that would
242 otherwise be either sparsely connected or disconnected [23]. Thus, betweenness
243 centrality an important centrality measure for a social, technological, computer, and
244 biological networks. The higher efficiency of the strategy based on node betweenness
245 centrality with respect to the attack based on node rank in real-world networks can be
246 explained by the fact that in real-world networks some of the critical nodes (i.e. nodes
247 whose persistence strongly contribute to maintaining network integrity) are either not
248 highly linked, or that the highly-linked nodes are not located in the network core [23].

249 When applying the recalculated method, the newly-introduced *Combined* attack strategy
250 was the most efficient strategy to decrease *LCC* size in the scale free network configuration
251 model and in the Barabasi-Albert model up to $q = 0.1$. The *Combined* attack first select
252 nodes according to their rank, then, in the case of ties (i.e. nodes with the same rank), it

253 sequentially removes nodes according to their second degree. On the contrary, in the case
254 of ties *First* randomly chooses the removal sequence for the nodes with the same rank.
255 Thus, at the beginning of the attack to the network, when two or more major hubs have
256 the same number of links to other nodes, removing first the hub with the greatest second
257 degree causes a faster decrease in *LCC* size than to randomly select the removal sequence
258 for those hubs.

259 Later in the attack sequence, the *Combined* strategy was less efficient than the *First* strategy
260 to attack scale free networks; this might be due to the fact that after a certain fraction of
261 hubs has been deleted, removing first (in the case of ties) the node(s) with the highest
262 second degree(s) would remove more peripheral and less important nodes, i.e. nodes that
263 are less likely to be part of the largest connected component.

264 Further, the efficiency of attack strategies changed along the sequential removal of nodes.
265 This was particularly clear for networks with power-law degree distribution. It follows
266 that the percolation threshold, i.e. the fraction of nodes removed for which the size of the
267 largest connected component reaches zero, might be for some networks little correlated
268 with the fraction of nodes to be removed in order to reduce the largest connected
269 component to a size greater than 0. This result has important implications for applied
270 network science and deserves further investigations. For example, in the case of
271 immunization strategies, choosing the attack strategy according to the percolation
272 threshold may be of little use when the goal is to reduce as much as possible the size of
273 *LCC* with just a few targeted immunizations. Lastly, the use *LCC* as a measure of the
274 efficiency of the network may not be appropriate for immune networks. Immune
275 networks, such as neural or lymphocyte networks, reveal a specific and non-trivial

276 architecture and they can display peculiar features when diluted. For this reason,
277 differently from what happens in other kind of systems, when in immune networks the
278 LCC decreases, the performance of the network can actually improve [27,28,29].

279

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- 352

353 Figure captions

354

355 **Figure 1.** Size of normalized LCC and the fraction q of nodes removed for non-recalculated
356 targeted attacks to model networks. Points are plotted every 20 nodes removed for networks with N
357 $= 500$ and $N = 1\ 000$, and every 200 nodes removed for $N = 10\ 000$.

358 **Figure 2.** Size of normalized LCC and the fraction q of nodes removed for non-recalculated
359 targeted attacks to real-world networks. Points are plotted every 50 nodes removed for *Email* and
360 *Immuno* networks, and every 200 nodes removed for *Gnutella*.

361 **Figure 3.** Size of normalized LCC and the fraction q of nodes removed for recalculated targeted
362 attacks to model networks. Points are plotted every 20 nodes removed for networks with $N = 500$
363 and $N = 1\ 000$, and every 200 nodes removed for $N = 10\ 000$.

364 **Figure 4.** Size of normalized LCC and the fraction q of nodes removed for recalculated targeted
365 attacks to real-world networks. Points are plotted every 50 nodes removed for *Email* and *Immuno*
366 networks, and every 200 nodes removed for *Gnutella*.

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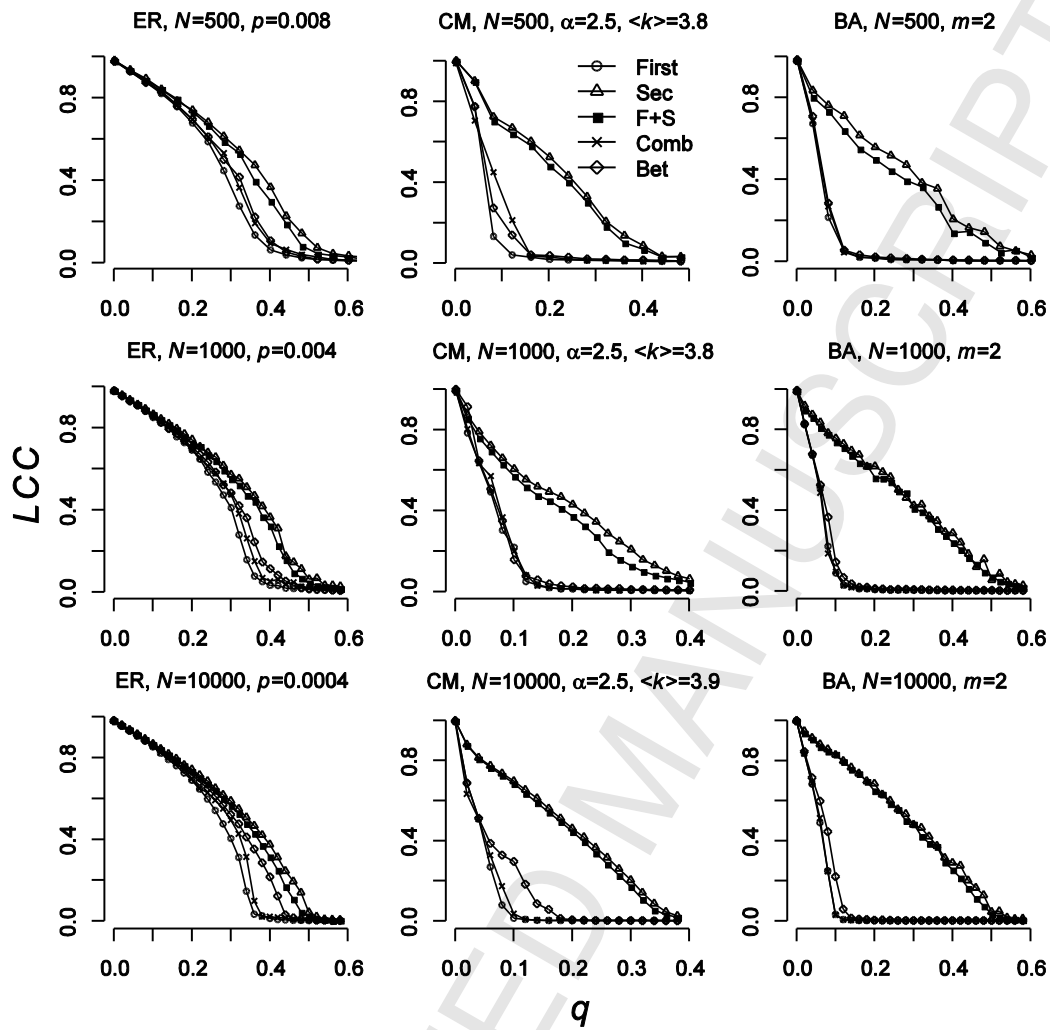
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Figure 1



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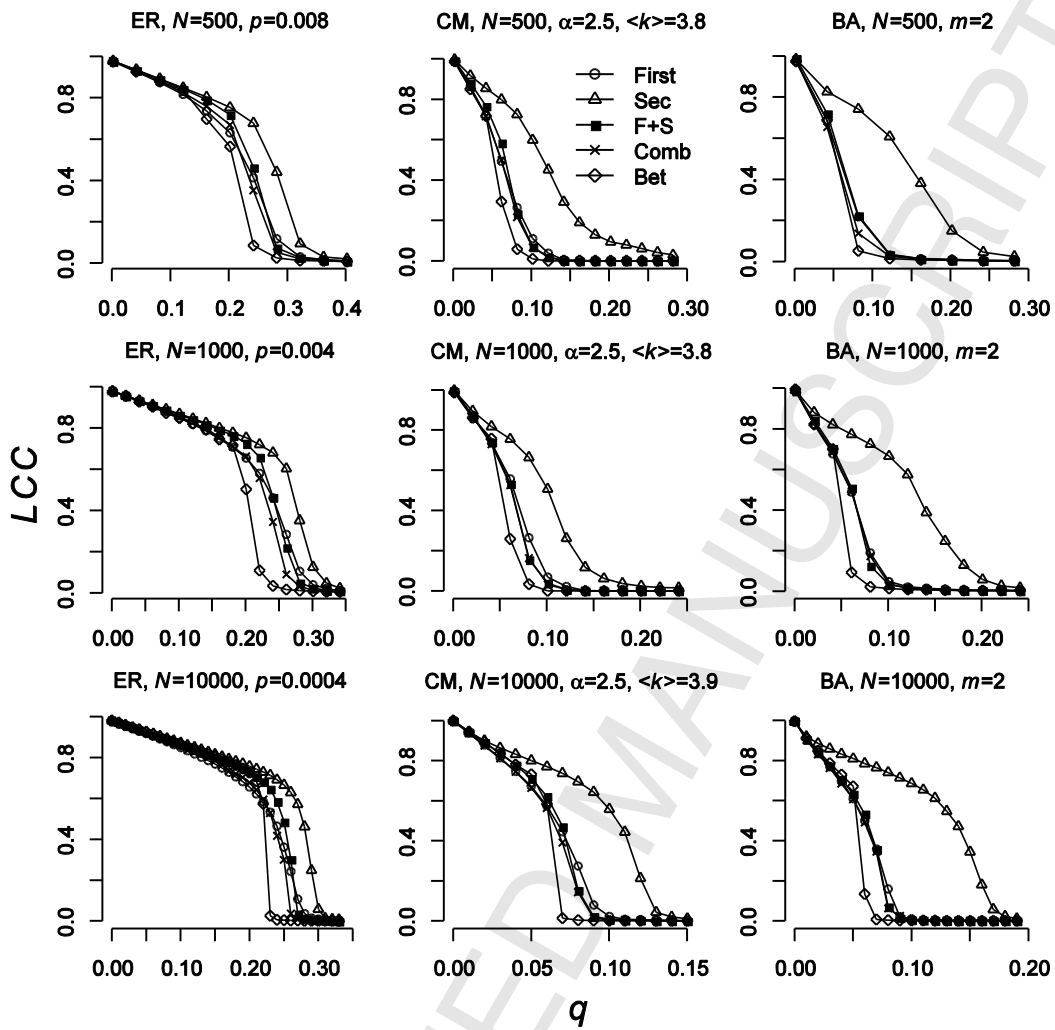
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Figure 2



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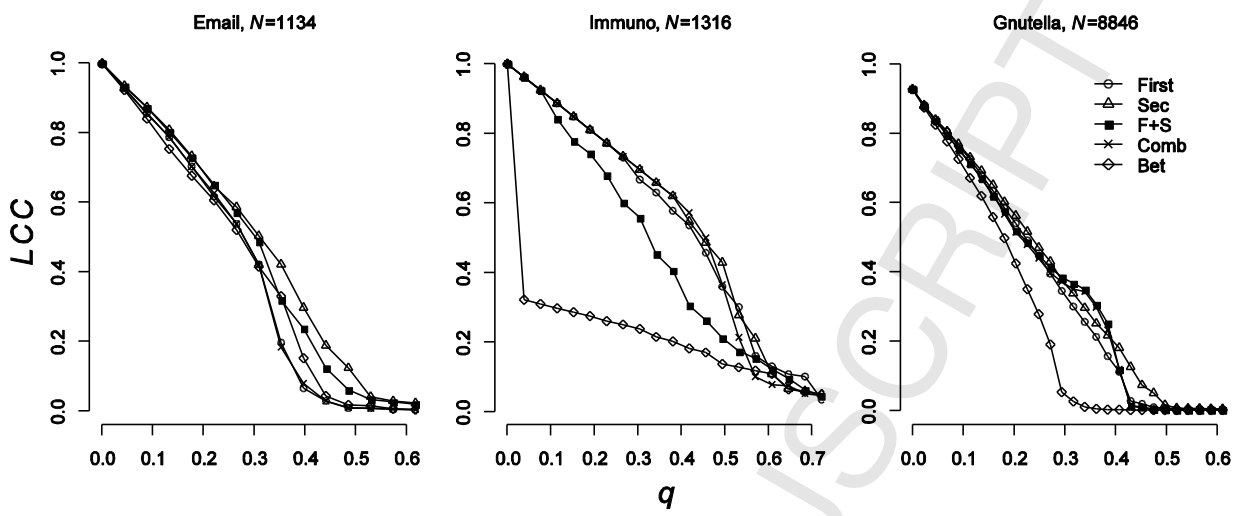
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Figure 3



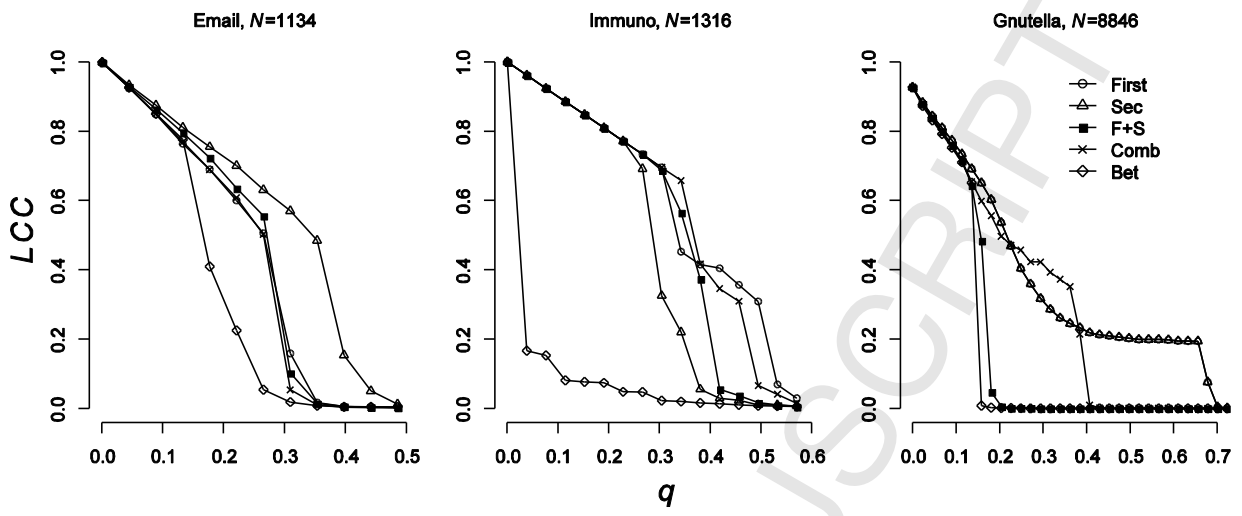
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Figure 4



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Highlights

We investigated the efficiency of network attack strategies

We used the size of the largest connected component as a damage measure

We tested 3 attack strategies introduced in this work for the first time

Deletion according to betweenness centrality was the most efficient attack strategy